THE CAUSE OF PLANETARY ROTATION ALSO A THEORY AS TO THE TAIL OF THE COMET

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Mr. Isaac T. Hinton,
% Mr. Thomas P. Woodward,
Hotel Sutter,
San Francisco, Cal.

Dear Sir:

Please accept our grateful thanks for the gift of your Cause of Planetary Rotation. I have placed one copy in the Observatory library and have kept one in the Main Library.

Yours very sincerely,

Librarian.

PLANETARY ROTATION

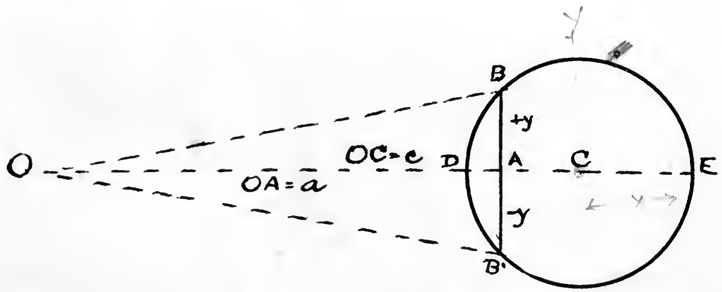
The following is an attempt to assign the cause of the moon always presenting the same face to the earth.

Take two small, equal masses, as indicated in the figure, at equal distances from the line of attraction. B attracts or is attracted by a force equal to $\frac{1}{a^2 + y^2}$, and B' by an equal force.

The components y and $-y$ are equal and opposite, their algebraic sum being equal to zero. The combined attraction of the two small masses in the direction OA is equal to—

$$2m.\frac{1}{a^2 + y^2} \cdot \frac{a}{(a^2 + y^2)^{1/2}} = \frac{2a}{(a^2 + y^2)^{1\frac{1}{2}}}$$

As either a or y increases, the attraction will diminish.



To find the attractive force of a circular disc, a resort must be had to calculus. Let BB' be the projection of the disc, and suppose y to be increased by a small quantity dy.

Then the differential of the attractive force of the disc will be $\frac{2\pi a y}{(a^2 + y^2)^{1\frac{1}{2}}} dy$. The integral of this is $\frac{-2\pi a}{(a^2 + y^2)^{1/2}}$, and tak-

ing this between the limits zero and y, we obtain the attraction of the circular disc, which is

$$2\pi\left(1 - \frac{a}{(a^2 + y^2)^{1/2}}\right) \quad . \quad . \quad . \quad . \quad . \quad (A)$$

This may be written

$$2\pi\frac{(a^2 + y^2)^{1/2} - a}{(a^2 + y^2)^{1/2}}$$

Multiplying both terms by $(a^2 + y^2)^{1/2} + a$, we obtain

$$\frac{2\pi y^2}{(a^2 + y^2) + a\,(a^2 + y^2)^{1/2}} = \frac{\text{Area of disc}}{\tfrac{1}{2}\,(a^2 + y^2) + \tfrac{1}{2}a\,(a^2 + y^2)^{1/2}}$$

This diminishes (y remaining the same) as a increases; therefore, if we had another disc on the other side of the center of the sphere and at the same distance from that center, the attraction of the off disc would be less than that of the near disc. Now, a sphere may be regarded as made up of an infinite number of such discs. Hence, the near hemisphere attracts with a greater force than the off hemisphere.

A more satisfactory proof of this, perhaps, is the following: When the disc is taken as a section of a homogeneous sphere, and c is the distance apart of the centers, the distance a becomes $c + x$ and y^2 becomes $r^2 - x^2$. Substituting these values in (A) we have

$$\pi\left(2 - \frac{2c + 2x}{(c^2 + 2cx + r^2)^{1/2}}\right)$$

If this be multiplied by dx, and the integral be taken between the limits $-r$ and $+r$, we shall have the attraction of the entire sphere. Dropping π for the present to simplify matters, we have

$$2dx - \frac{2cdx}{(c^2 + 2cx + r^2)^{1/2}} - \frac{2xdx}{(c^2 + 2cx + r^2)^{1/2}}$$

Integrating (using the formula $uv = \int udv + \int vdu$ for the last term) we have

$$2x - 2(c^2 + 2cx + r^2)^{1/2} - 2x\frac{(c^2 + 2cx + r^2)^{1/2}}{c}$$
$$+ \frac{2}{3c^2}(c^2 + 2cx + r^2)^{1\frac{1}{2}} \quad . \quad . \quad . \quad (B)$$

$$
\begin{array}{ll}
\text{Making } x = +r & \left\{ \begin{array}{l}
+2r - 2(c+r) \qquad -2r\left(\dfrac{c+r}{c}\right) + \dfrac{2(c+r)^3}{3c^2} \\[2ex]
-2r - 2(c-r) \qquad +2r\left(\dfrac{c-r}{c}\right) + \dfrac{2(c-r)^3}{3c^2} \\[1ex]
\hline
+4r - 4r \qquad -4r + \dfrac{12c^2r + 4r^3}{3c^2} = \dfrac{4\pi r^3}{3c^2}
\end{array} \right. \\
x = -r & \\
\text{Subtracting} &
\end{array}
$$

$$
\begin{array}{l}
(c+r)^3 = c^3 + 3c^2r + 3cr^2 + r^3 \\
(c-r)^3 = c^3 - 3c^2r + 3cr^2 - r^3 \\
\hline
\qquad\qquad 6c^2r \qquad\qquad + 2r^3
\end{array}
$$

That is to say, a homogeneous sphere attracts or is attracted as if its entire mass were concentrated at its center. (The geometrical demonstration of this truth given in the books is faulty.) But it is not a fact that the two hemispheres are attracted equally. To prove this, let us take the integral first from $-r$ to o, and then from o to $+r$. The integral (B) is:

$$
2x - 2(c^2 - 2cx + r^2)^{1/2} - 2x\frac{(c^2 + 2cx + r^2)^{1/2}}{c} + 2\frac{(c^2 + 2cx + r^2)^{1/2}}{3c^2}
$$

$$
\begin{array}{ll}
\text{Making } x = 0 & \left\{ \begin{array}{l}
0 - 2(c^2 + r^2)^{1/2} \qquad\qquad + 2\dfrac{(c^2 + r^2)^{1\frac{1}{2}}}{3c^2} \\[2ex]
-2r - 2(c-r) + 2r\dfrac{(c-r)}{c} \qquad + 2\dfrac{(c-r)^3}{3c^2} \\[1ex]
\hline
2c - 2(c^2 + r^2)^{1/2} - 2r\dfrac{(c-r)}{c} + 2\dfrac{(c^2 + r^2)^{1\frac{1}{2}}}{3c^2} \\[2ex]
\qquad\qquad - 2\dfrac{(c-r)^{1\frac{1}{2}}}{3c^2} \quad . \quad . \quad (a)
\end{array} \right. \\
x = -r & \\
\text{Subtracting} &
\end{array}
$$

Which, multiplied by π, is the attraction of the near hemisphere.

$$
\begin{array}{ll}
\text{Making } x = +r & \left\{ 2r - 2(c-r) - 2r\dfrac{(c+r)}{c} + 2\dfrac{(c+r)^3}{3c^2} \right. \\[2ex]
x = 0 & \quad 0 - 2(c^2+r^2)^{\frac{1}{2}} + \dfrac{2(c^2+r^2)^{1\frac{1}{2}}}{3c^2} \\[2ex]
\text{Subtracting} & \left\{ -2c + 2(c^2+r^2)^{\frac{1}{2}} - 2r\dfrac{(c+r)}{c} + \dfrac{2}{3c^2}\left[(c+r)^3 + (c^2+r^2)^{1\frac{1}{2}}\right] \quad . \; (b) \right.
\end{array}
$$

For the attraction of the off hemisphere. Adding (a) and (b), we obtain $\frac{4\pi r^3}{3c^2}$, as a check. To prove algebraically that (a) is greater than (b) would lead to complexity; hence a resort will be had to arithmetic, making $c = 12$ and $r = 5$. Substituting these values in (a), the result will be:

$$2c - 2(c^2+r^2)^{\frac{1}{2}} - 2r\frac{(c-r)}{c} + \frac{2}{3c^2}\left[(c^2+r^2)^{1\frac{1}{2}} - (c-r)^3\right]. \quad .(a)$$

$$24 - 2(169)^{\frac{1}{2}} - 10\frac{(7)}{12} \qquad + \frac{2}{432}\left[169^{1\frac{1}{2}} \qquad - 7^3\right]$$

$$24 \qquad - 26 \qquad - \frac{70}{12} \qquad + \frac{1}{216}\left[13^3 \qquad - 7^3\right]$$

$$
\begin{array}{r}
-2 \\
-5.833 \\
+8.583 \\ \hline
+\ .750
\end{array}
\qquad
\begin{array}{l}
12\ )\ 70\ (\ 5.833 \\
\quad\ \ 60 \\ \hline
\quad\ 100 \\
\quad\ \ 96 \\ \hline
\quad\ \ 40
\end{array}
\qquad
\begin{array}{r}
13^2 = 169 \\
13 \\ \hline
507 \\
169 \\ \hline
13^3 = 2197
\end{array}
\qquad
\begin{array}{r}
7^2 = 49 \\
7 \\ \hline
7^3 = 343
\end{array}
\qquad
\begin{array}{r}
13^3 = 2197 \\
7^3 = \ \ 343 \\ \hline
216\ )\ 1854\ (\ 8.583 \\
1728 \\ \hline
1260 \\
1080 \\ \hline
1800 \\
1728 \\ \hline
720
\end{array}
$$

And the attraction of the near hemisphere is .750.

For the off hemisphere the result will be:

$$-2c+2(c^2+r^2)^{\frac{1}{2}}-2r\frac{(c+r)}{c}+\frac{2}{3c^2}[(c+r)^3-(c^2+r^2)^{1\frac{1}{2}}] \quad (b)$$

$$-24+2(169)^{\frac{1}{2}} \qquad -10\frac{(17)}{12}+\frac{2}{432}[17^3 \qquad\qquad -13^3]$$

$$-24+26$$

	12) 170 (14.167	17	13	4913
	12	17	13	2197
	50	119	39	216) 2716 (12.574
	48	17	13	216
	20	289	169	556
	12	17	13	432
	80	2023	507	1240
	72	289	169	1080
— 24				
+ 26		$17^3=4913$	$13^3=2197$	1600
— 14.167				1512
+ 12.574				
+ .407				880
				864

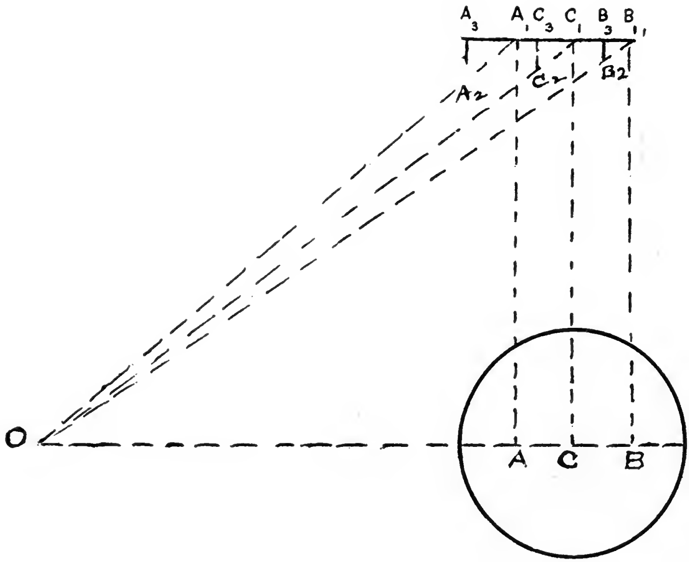
And the attraction of the off hemisphere is +.407.

$$\frac{4r^3}{3c^2}=\frac{500}{432}$$ Attraction of whole sphere.

432) 500 (1.157	Near hemisphere.............	.750
432	Off hemisphere..............	.407
680		
432	Whole sphere............	1.157
2480		
2160		
3200		
3024		

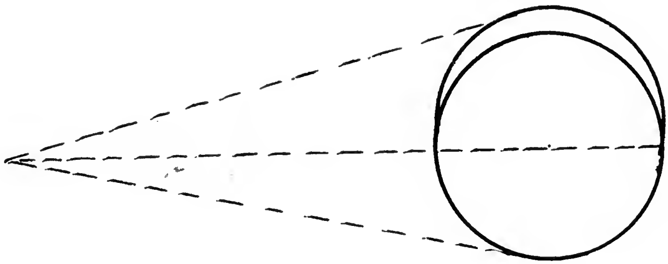
Now, suppose a body to be at the point C, and moving at right angles to OC, so that it will travel to C_1 in a certain interval of time, as indicated in the figure, C being the center of the sphere, and A and B being the centers of the mass of the respective hemispheres. If there were no force other than this tangential force, the center C would move to C_1. Suppose,

again, that afterwards an attractive force acts for the same length of time in the direction OC_1. If we regard the forces as acting on the whole sphere, the center will take some such position as C_2; and if we regard the forces as acting upon the two hemispheres separately, the points A and B will assume



the positions A_2 and B_2. The component distances AA_2, C_1C_2, B_1B_2, and A_2A_3, C_2C_3, B_2B_3 are not in the above ratio .750 : ½(1.157) : .407, though nearly so. However, the smaller the intervals of time and space are taken, the more nearly will the ratios approach equality, until, when the calculus limit is reached and the orbit becomes a curve, instead of a polygon, the lines *OA, OC, OB* will coincide within an infinitely small angle, and the lines A_3A_2, C_3C_2, B_3B_2 will attain the ratio .750 : ½(1.157) : .407. In other words, the original points *A, C, B* will assume the position $OA_2C_2B_2$, all on the same straight line. This clearly indicates a rotation going *pari passu* with the revolution. Hence, when gravity acts as a central acceleration, the secondary will always present the same face to the primary. Q. E. D.

The earth does not rotate according to this law, but here other forces have been or are now in action. The following is a highly improbable, but may be possible explanation: If the earth had as regards the sun no rotation at all, or a rotation (due to the moon's influence) once in about twenty-eight days, the face towards the sun would soon become very hot, and



the water on that side would evaporate rapidly and condense as ice on the surface of the earth's off hemisphere. After a while the condition would be somewhat as in the figure, the shaded portion representing ice.

Such a body would (I think) rotate with a continuous acceleration, the ratio of acceleration diminishing as the ice melted faster through the more frequent turning of the earth's face to the sun, until the acceleration became *nil*. The body would then continue to rotate at the speed attained when the acceleration became zero.

WEIGHING THE EARTH.

If a known weight were in a well at the point A in the figure on page 1, its attraction towards the center would be equal to the attraction of the mass BEB' less that of the mass BDB', which can be found by integration, and the attraction or weight of the earth could be found if we knew the weight of the spherical segment BDB'. This would be exceedingly difficult to obtain with any degree of accuracy on the land, but at sea we can go down five miles, and the weight of a spherical segment of water whose height is five miles can be calculated to a nicety.

THE TAIL OF THE COMET.

In endeavoring to explain this phenomenon by the laws of mechanics, the writer chanced upon the preceding demonstration, without making any progress on the main problem. However, he ventures the following explanation of the comet's tail from physical laws:

The nucleus of the comet, it appears to be admitted, is a mass of very hot vapor or vapors, and if this is true it must be surrounded by a vast mass of less hot and less condensed vapors, this large mass, or a portion thereof vastly larger than the nucleus, being somtimes faintly visible by starlight. The comet is therefore an illuminated nucleus surrounded by an immense envelope of vapor, hot, but not as hot as the nucleus. If a cannon-ball were lying on the surface of the earth, it would weigh more at 12 midnight than at noon, since in the one case the sun's attraction would be added to gravity and in the other it would be subtracted from it. The moon at conjunction would have a greater effect, perhaps enough to be noticed on a fine spring balance. So, the comet would be flattened on the side away from the sun, and elongated on the near side, thus assuming an egg-like form. However, when the distance is great, the form is probably nearly a sphere. Now, the nucleus of a comet is dense enough to cast a shadow, and against the dark background of that shadow the heated particles immediately surrounding that shadow become visible, while all the rest of the vast volume of the comet except the nucleus fail to give out sufficient light to be visible. Their light is obscured just as that of the moon is obscured in the daytime, while the particles around the shadow of the nucleus are more bright from the same reason that the corona of the sun appears brighter during an eclipse. The length of the tail depends on the heat of the comet, the distance from the sun and the angle of view from the earth. The curvature is due to the difference in time taken by the light coming from the nucleus and from the tail. It is most noticeable when the comet is near the sun, since then the velocity of the nucleus is immense, and that of the end of its shadow is very much more so.